Reg. No. | | | | | | | | |

G. VENKATASWAMY NAIDU COLLEGE (AUTONOMOUS), KOVILPATTI – 628 502.

PG DEGREE END SEMESTER EXAMINATIONS - APRIL 2025.

(For those admitted in June 2023 and later)

PROGRAMME AND BRANCH: M.Sc., MATHEMATICS

SEM	CATEGORY	COMPONENT	COURSE CODE	COURSE TITLE
II	PART-III	CORE ELECTIVE-3	P23MA2E3B	MATHEMATICAL STATISTICS

Date & Session: 30.04.2025/AN **Time : 3 hours** **Maximum: 75 Marks**

Course Outcome	Bloom's K-level	Q. No.	**SECTION – A (10 X 1 = 10 Marks)** Answer ALL Questions.
CO1	K1	1.	If X and Y are independent poisson variates then the conditional distribution of X,given X+Y is. a) poisson b) binomial c) normal d) chisquare
CO1	K2	2.	The random variables X_1 and X_2 are said to be stochastically independent iff $f(x_1,x_2)$=. a) $f_1(x_1)$ b) $f_2(x_2)$ c) $f_1(x_1)\ f_2(x_2)$ d) $f_1(x_2)$
CO2	K1	3.	If $(1-2t)^{-6}, t<\frac{1}{2}$ is th moment generating function of a random variable then its variance is. a) 3 b)12 c) 24 d) 5
CO2	K2	4.	If X_i (i=1,2,…n) denote a random sample from $n(\mu,\sigma^2)$, then $Y=\sum_1^n(\frac{X_i-\mu}{\sigma})^2$ has – distribution. a) normal b) cauchy c) beta d) chi-square
CO3	K1	5.	If $X_i=i$,i=1,2,…n then the value of $\bar{X}=\frac{\sum X_i}{n}$ is. a) $\frac{n+1}{2}$ b) $\frac{n^2-1}{12}$ c) $\frac{n-1}{2}$ d) $\frac{n^2+1}{12}$
CO3	K2	6.	If F has an F distribution with parameters r_1 and r_2 then $\frac{1}{F}$ has an F distribution with parameters. a) $\frac{r_1}{r_2}$ b) $r_1.\ r_2$ c) r_2 and r_1 d) $\frac{1}{r_2}$
CO4	K1	7.	If $X_1, X_2 \ldots\ X_N$ is a random sample from a distribution with m.g.f M(t) then the m.g.f of $\sum_1^n X_i$ is a) $[M(\frac{t}{n})]^n$ b) M(t) c) $[M(t)]^n$ d) $M(\frac{t}{n})$
CO4	K2	8.	The sum of n mutually stochastically independent normally distributed variables has a --- distribution a) normal b) exponential c) poisson d) binomial
CO5	K1	9.	Let $\bar{X}$ denote the mean of a random sample of size 128 from a gamma distribution with α=2 and β=4 then the variance of $\bar{X}$ is a) $\frac{8}{128}$ b) $\frac{1}{2}$ c) $\frac{3}{128}$ d) $\frac{1}{4}$
CO5	K2	10.	Let $\overline{X_n}$ and S_n^2 denote respectively, the mean and the variance of a random sample of size n from a distribution that is $n(\mu,\sigma^2)$, $\sigma^2>0$ then $\frac{\sigma X_{\bar{n}}}{S_n}$ ---- to μ a) converge stochastically b) both (a) and (b) c) diverge stochastically d) neither (a) nor (b)

Course Outcome	Bloom's K-level	Q. No.	**SECTION – B (5 X 5 = 25 Marks)** Answer ALL Questions choosing either (a) or (b)
CO1	K2	11a.	Let X denote the random variable with E(X)=3 and $E(X^2)$=13 then find the lower bound for $P_r(-2< x <8)$ using chebyshev's inequality. **(OR)**

CO1	K2	11b.	Let X_1 and X_2 have the joint p.d.f f(x_1,x_2)=$\frac{x_1+2x_2}{18}$ where (x_1,x_2) =(1,1),(1,2),(2,1),(2,2) and zero elsewhere.Find the marginal p.d.f of X_1 and X_2.Also find P_r(X_1=3) and P_r(X_2=2).
CO2	K2	12a.	Derive the m.g.f of gamma distribution and hence find the mean and variance of the distribution **(OR)**
CO2	K2	12b.	Let X be n(μ,σ^2) so that P_r(X< 89)=0.90 and P_r(X< 94)=0.95.Find μ and σ^2
CO3	K3	13a.	Show that $S^2=\frac{1}{n}\sum(X_i-\overline{X})^2=\frac{1}{n}\sum X_i^2-\bar{X}^2$ where $\bar{X}=\frac{1}{n}\sum X_i$ **(OR)**
CO3	K3	13b.	Let X denote the p.d.f f(x)=1,0< $x<1$,zero elsewhere.Show that the random variable Y=-2logX has a chi-square distribution with 2 degrees of freedom.
CO4	K3	14a.	Let X_i (i=1,2,...n) denote a random sample of size n from n(μ,σ^2).Prove that Y=$\sum_1^n(\frac{X_i-\mu}{\sigma})^2$ has a chi-square distribution with n degrees of freedom. **(OR)**
CO4	K3	14b.	If $\bar{X}$ is the mean of a random sample of size n from a normal distribution with mean μ and variance 100. Determine n so that $P_r(\mu-5<\bar{X}<\mu+5)=0.954$
CO5	K4	15a.	Compute an approximate probability that the mean of a random sample of size 15 from a distribution having p.d.f f(x)=$\begin{cases}3x^2, & 0<x<1\\ 0, & elsewhere\end{cases}$,is between $\frac{3}{5}$ and $\frac{4}{5}$ **(OR)**
CO5	K4	15b.	Let Y_n denote a random variable which is b(n,p),0< $p<1$,then prove that $U_n=\frac{Y_n-np}{\sqrt{np(1-p)}}\sim$ n(0,1).

Course Outcome	Bloom's K-level	Q. No	<u>SECTION – C</u> (5 X 8 = 40 Marks) Answer <u>ALL</u> Questions choosing either (a) or (b)
CO1	K4	16a.	State and Prove chebyshev's inequality **(OR)**
CO1	K4	16b.	Let f(x,y) =2,0 < $x<y$ <1,zero elsewhere be the joint p.d.f.of X and Y.Show that the correlation coefficient between X and Y is $\frac{1}{2}$
CO2	K5	17a.	Show that $\iint_\mu^\infty \frac{1}{\Gamma(k)}z^{k-1}e^{-z}dz=\sum_{x=0}^{k-1}\frac{\mu^x e^{-\mu}}{x!}$,k=1,2 ,... **(OR)**
CO2	K5	17b.	Let X and Y have a bivariate normal distribution with paramters μ_1=3 and μ_2=1,δ_1^2=16, δ_2^2=25 and $\rho=\frac{3}{5}$.Determine the following probabilities a.P_r(3< y <8) b. P_r(3< y <8/x=7) c. P_r(-3< y <3) d. P_r(-3< y <3/y=-4)
CO3	K5	18a.	Let the random variable X have the p.d.f f(x)=1,0< $x<1$ and zero elsewhere and let X_1 and X_2 denote a random sample from this distribution .Find the joint p.d.f of the two random variables $Y_1=X_1+X_2$ and $Y_2=X_1-X_2$.Find also the marginal p.d.f s of Y_1 and Y_2. **(OR)**
CO3	K5	18b.	Derive Student's 't' distribution
CO4	K5	19a.	Let $Y_1<Y_2$ denote the order statistics of a random sample of size 2 from n(0, σ^2).Show that E(Y_1)=$\frac{-\sigma}{\sqrt{\pi}}$ and E(Y_2)=$\frac{\sigma}{\sqrt{\pi}}$ **(OR)**
CO4	K5	19b.	Let X_i (i=1,2,...n) denote a random sample of size n from n(μ,σ^2).Show that E(S^2)=(n-1)$\frac{\sigma^2}{n}$,where S is the variance of the random variable
CO5	K6	20a.	Let S_n^2 denote the variance of a random sample of size n from n(μ,σ^2).Prove that $\frac{nS_n^2}{n-1}$converges stochastically to σ^2 **(OR)**
CO5	K6	20b.	State and Prove Central limit theorem